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Scaling of Growth Rate Volatility for Six Macroeconomic Variables

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ABSTRACT

We study the annual growth rates of six macroeconomic variables: public debt, public health expenditures, exports of goods, government consumption expenditures, total exports of goods and services, and total imports of goods and services. For each variable, we find (i) that the distribution of the growth rate residuals approximately follows a double exponential (Laplace) distribution and (ii) that the standard deviation of growth rate residuals scales according to the size of the variable as a power law, with a scaling exponent similar to the scaling exponent found for GDP [Economics Letters 60, 335 (1998)]. We hypothesise that the volatility scaling we find for these GDP constituents causes the volatility scaling found in GDP data.

KEY WORDS:

composite index, complex phenomena, dynamics

JEL Classification: C1, O1

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Introduction and Data Analysis

Volatility scaling is an important factor in describing the relationship between the “micro” and “macro” levels. In particular, the way volatility changes under different measurement scales ties the microstructure of a given system to its macroscopic observables through scaling laws. Therefore, empirical studies of volatility scaling may provide better insight into the fundamental processes governing systems at the “micro” level, which then produce the observed patterns of scaling at the “macro” level.

The study of volatility scaling has been applied to different levels of aggregation in macroeconomics,

ranging from the “micro” level of company products (see, e.g., Growiec et al., 2008) to the “macro” level of countries (see, e.g., Canning et al., 1998, Podobnik et al., 2008). For countries, Barro (1991) assumed the existence of heteroscedasticity in growth rates of per capita real gross domestic product (GDP). Head (1995) argued that the higher GDP variances of smaller countries can be explained by their open economies. However, the exact functional dependence between the volatility of GDP growth rates and country size was not understood until Canning et al. (1998) and Lee et al. (1998) found that the standard deviation of the logarithmic growth rate, R , decreases with increasing GDP (i) as a power law, $\sigma(R) \sim (GDP)^\beta$, with a scaling exponent of $\beta \approx -0.15$. For countries, researchers have also found that the pdf, $P(R)$, of the GDP growth rate (ii) has a Laplace (double exponential) form in

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its central part. Recently, Fu et al. (2005) found that (iii) power laws exist in the tails of $P(R)$. These results, obtained from macroeconomic data, are consistent with results obtained from microeconomic data, such as the number of employees and company sales (see, e.g., Stanley et al. 1996). Regarding (iii), Podobnik et al (2011) reported that an asymmetric Levy distribution, which has power-law tails and is characterised by infinite variance, is a good model for several multiple-credit ratios that are used in financial accounting to quantify a firm's financial health, such as the Altman Z score (1968). The asymmetric Levy distribution also models changes in individual financial ratios.

The expenditure method is the most common way to calculate a country's GDP. In this method, GDP is calculated as the sum of five macroeconomic variables, including exports, imports, and government consumption expenditures. We identify patterns of volatility scaling for these three GDP constituents and hypothesise that volatility scaling in these factors contributes to volatility scaling observed in a country's GDP. We also show that similar volatility scaling exists for three other macroeconomic variables.

We analyse the scaling of annual growth rates, $R_{a,t} \equiv \ln(S_{a,t+1}/S_{a,t})$, that are calculated for six macroeconomic variables, $S_{a,t}$, where $a=1$ denotes public debt, $a=2$ denotes government consumption expenditures, $a=3$ denotes public health expenditures, $a=4$ denotes exports of goods, $a=5$ denotes total imports of goods and services, and $a=6$ denotes total exports of goods and services. We analyse data for a range of different countries, I , and different years, t . We define the growth rate residuals, $r_{a,t}$, as

$$R_{a,t} \equiv \mu_a + r_{a,t}, \quad (1)$$

where μ_a is the expected growth rate of S_a in country I . As the number of data points for each country is limited, we combine all $(r_{a,t}, S_{a,t})$ pairs for each variable S_a into a common data set to increase statistical power. From Table 1, we see that for each of the variables S_a , skewness and kurtosis deviate from the values expected under a Gaussian distribution (0 and 3, respectively).

Public debt data, available at Inter-American Development Bank (<http://www.iadb.org/RES/databases.cfm>), refer to gross central government debt. All other

data are found at The World Resources Institute (<http://earthtrends.wri.org>).

In Section 2, we find that the residuals, $r_{a,t}$, are not normally distributed but are exponentially distributed. This result may have useful implications for our study. For example, the Schwarz Information Criterion (see Schwarz, 1978), often proposed as a statistical criterion for model selection, requires that data follow an exponential distribution. In Section 3, we use a rigorous statistical approach. For each of the six macroeconomic variables and total labor force, we find that the standard deviation of the growth rate residuals, $\sigma(r_a)$, follow a power law with the size of variable S_a . Thus, the two findings (i) and (ii) from above imply that for the six macroeconomic variables analysed, r_a are neither normally distributed nor homoscedastic.

Graphical approach

Next, we investigate whether $\sigma(r_a)$ depends on the size of macroeconomic variable S_a . First, we qualitatively explain the growth rate for each macroeconomic variable, a . We sort the data set for each a into three subsets of equal size (small, medium, and large S_a).

We plot the empirical pdf of the residuals for the smallest and largest subsets for public debt [$a=1$ in Fig. 0(a)] and government consumption expenditure [$a=2$ in Fig. 0(b)]. The pdfs are plotted on a linear-log scale to emphasise that the absolute value of the residuals is double exponential (Laplace). If r_a were normally distributed, then the pdfs would be parabolas.

Next, we argue qualitatively that for each variable S_a , r_a are heteroscedastic because the standard deviation $\sigma(r_a)$ varies with the size of S_a . In Figs. 1(a) and 1(b), we see that for $a=1$ and $a=2$, the r_a obtained for countries with large S_a values have a smaller $\sigma(r_a)$.

Quantitative analysis

Least Squares Regression

Next, we quantitatively investigate how the volatility of growth rates changes with the size of S_a . For each S_a , we partition the entire sample into ten equal subintervals of $\log S_a$. Then, in Fig. 2(a), we plot the standard deviation, $\sigma(r_a)$, of the growth rate residuals, r_a , versus the size of S_a in the corresponding interval for each macroeconomic variable S_a . From Fig. 2(a), we find that

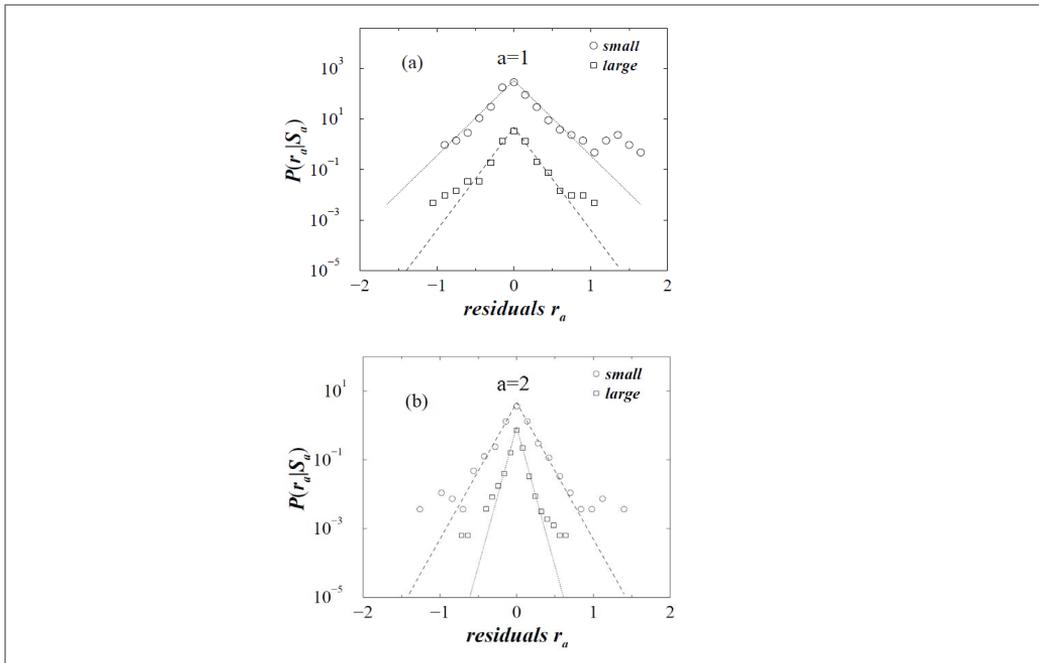


Figure 1. Conditional probability distributions $P(r_a|S_a)$ of the logarithmic growth rate residuals r_a of (a) public debt ($a=1$) and (b) government consumption expenditures ($a=1$) for two different ranges of S_a

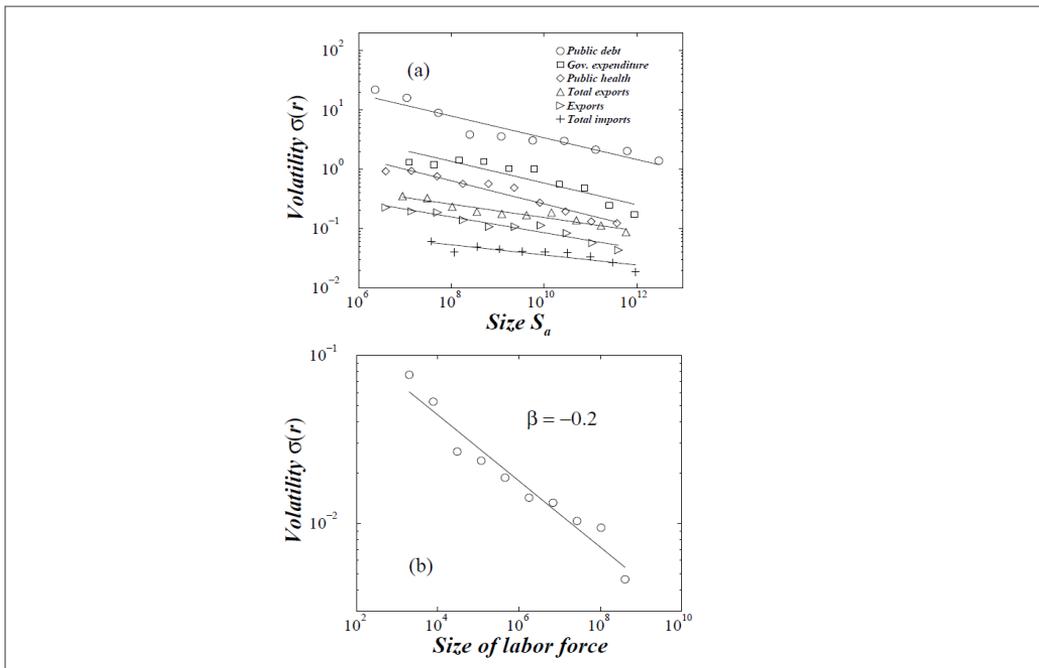


Figure 2. (a) Standard deviation $\sigma(r_a)$ of the one-year logarithmic growth rate residuals r_a as a function of the average value of S_a for all six macroeconomic variables. (b) Standard deviation $\sigma(r_a)$ of the one-year logarithmic growth rate residuals r_a as a function of the average value of labor force.

Table 1. Skewness, kurtosis, and power-law exponent. The three macroeconomic variables denoted by (*) are GDP constituents.

	Public Debt	Gov. Consumption*	Public health exp.	Total Exports*	Exports	Total imports*
skewness	3.06	0.46	0.63	-0.30	-0.11	-0.36
kurtosis	44.10	22.23	13.12	11.13	20.90	7.13
β_a	-0.18 ± 0.04	-0.18 ± 0.06	-0.20 ± 0.03	-0.11 ± 0.02	-0.13 ± 0.02	-0.09 ± 0.03

Table 2. Maximum likelihood estimates of the six macroeconomic variables. Macroeconomic variables denoted by (*) are GDP constituents. In the parantheses, we indicate

Error	Estimate of N	Estimate of β	Log Likelihood
Public Debt (2805)			
I Gaussian	0.24		50.44
II Gaussian	4.53	-0.14	506.82
III Laplace	0.19		902.15
IV Laplace	1.58	-0.09	987.75
Government consumption expenditure* (5819)			
I Gaussian	0.15		2959.88
II Gaussian	2.74	-0.14	3399.60
III Laplace	0.13		3876.15
IV Laplace	1.83	-0.13	4087.04
Public health expenditure (695)			
I Gaussian	0.17		239.69
II Gaussian	2.74	-0.15	323.84
III Laplace	0.16		348.85
IV Laplace	1.60	-0.12	379.63
Total Exports* (4269)			
I Gaussian	0.19		1116.28
II Gaussian	1.04	-0.08	1274.16
III Laplace	0.19		1421.14
IV Laplace	0.90	-0.07	1484.03
Exports (goods) (7314)			
I Gaussian	0.26		-403.60
II Gaussian	2.23	-0.11	149.36
III Laplace	0.23		815.15
IV Laplace	2.13	-0.11	1090.85
Total Imports* (4269)			
I Gaussian	0.17		1508.97
II Gaussian	0.56	-0.05	1561.06
III Laplace	0.18		1591.47
IV Laplace	0.41	-0.04	1604.55

for each S_a , $\sigma(r_a)$ decreases with S_a as a power law:

$$\sigma(r_a) \sim N_a (S_a)^{\beta_a}. \tag{2}$$

Using the method of least squares regression for each S_a , we estimate the parameters N_a and β_a for the regression $\ln \sigma(r_a) = \ln N_a + \beta_a \ln S_a$. The results are shown in Table 1. Surprisingly, the scaling exponents, β_a , are within the confidence interval of the scaling exponent, $\beta = -0.15 \pm 0.03$, reported for GDP (Canning et al., 1998).

The classical regression model, along with many other models in economics, assumes a normal distribution of residuals and assumes that variance remains constant as the size of the variable increases. When the residuals obey the latter assumption, they are said to be homoscedastic. We show that for each of the macroeconomic variables studied, the residuals are neither normally distributed nor homoscedastic.

Labor force is another key macroeconomic variable. Hence, we also plot $\sigma(r_a)$ for labor force [Fig. 2(b)]. Interestingly, we find this macroeconomic variable has residuals that are normally distributed. However, $\sigma(r)$ decreases with the size of the labor force, S , as a power law with an exponent, similar to the exponential values describing the six macroeconomic variables analysed above.

Maximum likelihood estimation

The maximum likelihood (ML) estimators are model parameters that have the maximum likelihood of generating the given sample. We again assume $\ln \sigma(r_a) = \ln N_a + \beta_a \ln S_a$. We shall test the hypothesis that the growth rate residuals are either normally distributed

$$P(r_a | S_a) = \frac{1}{\sqrt{2\pi}\sigma(r_a)} \exp(-r_a^2/2\sigma(r_a)^2), \tag{3}$$

or double exponentially (Laplace) distributed

$$P(r_a | S_a) = \frac{1}{\sqrt{2}\sigma(r_a)} \exp(-\sqrt{2} |r_a| \sigma(r_a)) \tag{4}$$

Table 2 shows our estimates of the parameters obtained in Eqs. (3) and (4) by employing the ML approach.

First, we observe from Table 2 that the log likelihood increases when the assumption of normal residuals (I and II) is replaced by the assumption of double ex-

ponential residuals (III and IV). Furthermore, we find that the log likelihood increases when the assumption of constant standard deviation of the residuals (I and III) is replaced by the assumption that there is a power-law dependence between the standard deviation $\sigma(r_a)$ and the size of variable S_a (II and IV). When constant standard deviation is replaced by the power-law dependent standard deviation for the residuals, we move from the estimation of one parameter (N_a) to the estimation of two parameters (N_a and β_a), where the second parameter (β_a) can be either genuine or spurious, depending on the statistical significance. When the log likelihood approach is employed to test the statistical significance of the model's parameters, adding a spurious parameter results in a twofold increase in log likelihood. For large samples, this parameter follows a χ^2 distribution with one degree of freedom. We accept that the additional parameter is significantly different from zero if a twofold increase in log likelihood is larger in magnitude than the 95% critical value, 3.84, for the χ^2 distribution. From Table 2, for each variable S_a , the increase in the log likelihood is much larger than the critical value of 3.84 when we assume power-law dependence for the standard deviation instead of a constant standard deviation. For example, for public debt under the assumption of double exponential residuals, twice the difference in the log likelihood is 171.2; thus, we reject the hypothesis that $\beta = 0$. We conclude that for each variable analysed, the power-law dependence between the standard deviation, $\sigma(r_a)$, and the size of the variable, S_a , are statistically significant.

Conclusions

By analysing many macroeconomic variables, we reject the microeconomic-level hypothesis that a country's economy is composed of entities with identically distributed Gaussian residuals. We hypothesise that the volatility scaling we find in GDP constituents results in the volatility scaling found in GDP data. Our finding that residuals for a broad range of macroeconomic variables are neither normal nor homoscedastic restricts the set of microeconomic variables that can be used to generate observed patterns of macroeconomic scaling (see Wu et al. 2001). In the maximum likelihood approach, when the number of model parameters is increased, researchers commonly employ a statistical criterion for model selection. The BIC or Schwarz Information Crite-

tion is an asymptotic result that is derived under the assumption that data follows an exponential distribution. We show that this assumption holds for many different macroeconomic variables.

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