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Static and Dynamic Price Effects Motivated by Innovation and Imitation: Novel Insights Using the Barone's Curve

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ABSTRACT

Luigi Barone's famous curve offers an excellent framework for the study of the microeconomic and macroeconomic implications of innovation and imitation. However, neither Barone nor his epigones have been able to sufficiently "exploit" his contribution to date. Complementing his analysis of supply (covering unit costs and marginal costs of production) to the forces of aggregate demand provided by the macroeconomic aggregate demand–aggregate supply model (AS-AD) analysis would be required in order to identify the determinants of the equilibrium price level in the economy. Moreover, a dynamic interpretation (provided by an inhomogeneous difference equation of the second order) of Barone's key economic growth factors (innovation and imitation) makes it easier to identify the cyclical properties of the macroeconomic price changes. These cyclical price movements have proven to be empirically relevant in the case of Germany (2000–2017), while patent record (as an indicator for the occurrence of innovation) appears to follow a random walk (Germany, 2000–2017).

KEY WORDS: Innovation, imitation, Barone's Curve, static and dynamic modeling of macroeconomic price (change) formation

JEL Classification: B31, B22, E32, O31, O11, O47

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1. Introduction

Digitalization and artificial intelligence have emerged as the keywords in the actual discussion regarding the novel trends in productivity increase. No matter what the means are, it appears to be undisputed that innovation and imitation remain the driving forces behind the market-oriented supply strategies of competitive firms. These strategies, along with the respective forces of demand, are able to explain the equilibrium static market price. In this context, it is worth recalling and further developing a concept, which was originally presented by Enrico Barone (see also Bradley & Mosca, 2012). Barone was an Italian economist who was born in 1851 and died in 1924. He became famous for developing the so-called Barone's Curve (Barone, 1935, pp. 19–21), named indeed after the author. The objectives of this report are the following: in the next section, the original contribution of Luigi Barone is verbally and graphically explained. The next section complements, and thereby extends Barone's contribution to determining the static equilibrium price level. In the fourth section, we present a dynamic macroeconomic interpretation of Barone's Curve, which would enable us to determine the

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cyclical price effects of innovation and imitation. In the fifth section, empirical evidence regarding the dynamics of price changes and the determinants of patent record (as a indicator for innovation activity) in the case of Germany (2000–2017) are presented. Next, in the sixth section, some policy implications are addressed and we provide certain tentative conclusions. Sections 7 and 8 are devoted to present the bibliographic references we used and a mathematical annex, respectively.

2. The Original Barone's Curve Framework

In the original Barone's Curve (Figure 1), all firms are ranked from left to right according to their minimum unit costs of production and an upward stepping curve is achieved when all capacities of production are ordered in a line with their respective unit costs. It holds for each firm that its marginal costs curve intersects its respective unit cost curve at its respective minimum. The individual supply is then determined by the intersection between the firm's own marginal costs with the horizontal price line. By aggregating all individual marginal cost curves, we obtain the total supply curve. Therefore, the amount of market production is determined as the result of the intersection of this supply curve with the price line (Helmstädter 1986b, pp. 70f.).

Suppose initially that, for a given price P, the intersection of the Barone's Curve with this price level determines the actual size of the output. The intersection point on the Barone's Curve is necessarily associated with a point on the aggregate marginal cost curve. Thus, it yields the exact point where a marginal supplier may cover his/her unit costs with his/her revenue per unit of production, which is directly given by the price. Net earnings per unit of production are hence zero. This implies as well that all former (i.e., more efficient) suppliers on the market are able to earn positive profits per unit of production. In Figure 1, this fact is illustrated by the striped area below the price line and above the Barone's Curve (i.e., the aggregate unit costs of production curve). This area is called the "total economy difference profit".

The area below the Barone's Curve can be interpreted as the sum of all cost factors. What might be the possible reasons for the existence of different unit costs of production? Barone himself pointed out the different capacities for innovation distributed within the firms. The entrepreneurs, who are able to introduce innovations into their technologies of production, will get benefits and hence be able to produce at lower costs than their competitors. Consequently, substantial gaps will appear among the profits of the suppliers. Innovation, hence, will always tend to lower the unit cost of production and accordingly increase the level of profit per unit of production. The suppliers, who gain cost advantages due to the effects of innovation, will, therefore, get differences in profits. A special case arises when all firms exhibit identical cost structures or employ identical technologies of production. Only in this special case, the unitary costs of the production will be identical for all firms and the Barone's Curve will be drawn as a horizontal line, parallel to the abscissa.

Helmstädter (1986b, pp. 73–76) further interpreted the original Barone's Curve with respect to circular, total economy effects. In a closed economy without a government, the total unit costs of production may work well as a proxy for labor unit costs. The area below the Barone's Curve then equals the total wage sum (which also stands for total private consumption when workers do not save) and the striped area called difference profits represents total investment and savings in the economy under the assumption that profit earners only save and they do not consume.

As shown by Sell and Ostermair (2018, p. 972 ff.), there may exist a proportional relation between the overall costs per unit of production on the one side, and the aggregate marginal costs on the other side. This possibility hinges upon the presence of a linear-homogeneous production function, as given by the Cobb-Douglas-Function. In such a case, the profit quota remains unaltered (and hence the wage quota as well) and corresponds to the production elasticity of capital (or labor). This assumption cannot be applied to the Barone's Curve analysis, thus we may deal here with a flexible distribution of incomes (profits, wages).

3. Explaining the Static Price Level Extending the Original Barone's Curve Framework

Meyer's models (1989) and other similar and relevant contributions that refer to Barone, such as Iwai's (1984a, 1984b), are surprising since they do not discuss the issue of the price level and its factors of change. Instead, they simply assume that "the price is equal

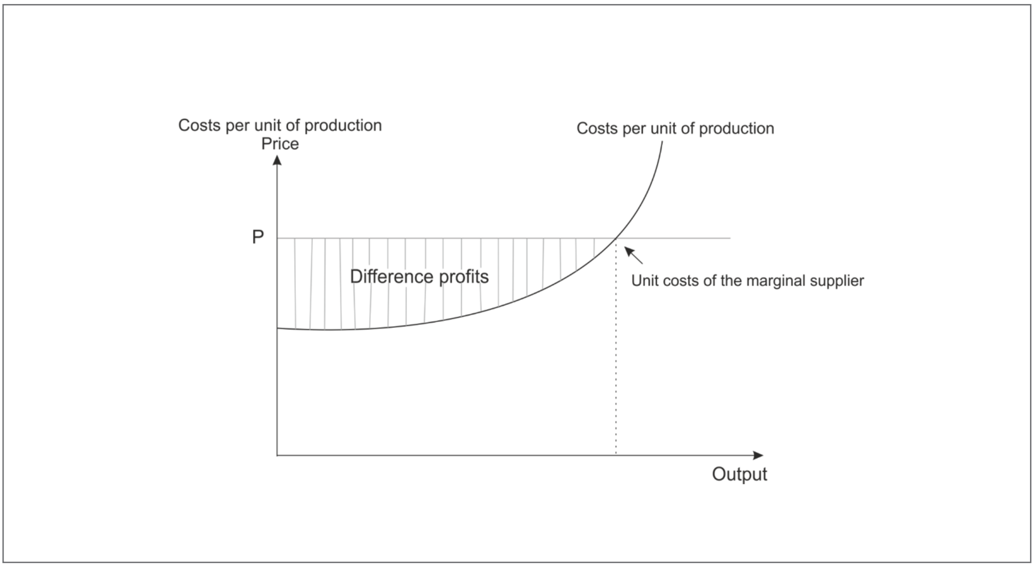

Figure 1. The original Barone's Curve.
Sources: Blümle, 1989; Külp 2017; own compilation.

for all firms" (Meyer 1989, p. 315). More specifically, Helmstädter (1986b) even uses the term of "a given price level" (Helmstädter 1986b, p. 73). In his model, he determines the price level assuming that the unitary costs of the marginal supplier are exactly covered (Helmstädter 1986b, p. 74). However, is it not actually true that this assumption stands for any price level? Is it not true as well that we always have to deal with a marginal supplier whose unitary costs are exactly covered by the actually ruling price level? Despite the fact that with this kind of method aggregate demand is totally neglected, the interdependence of demand and supply is essential for the explanation of the price level and its changes (see below).

In our own approach, we lean on the principle of horizontal competition (see Engelkamp & Sell, 2017, pp. 28–29) in a market economy, in addition to other tools. According to this principle, high prices are a strong indicator, for which special profit opportunities arise. However, saying this is not the same as arguing for a slope of profits achievable for innovators (see Blümle 1989, p. 21). Moreover, high (and hence above average) prices will tend to attract innovators into the respective market. This is because their ability to improve existing technologies of production that lower the average and marginal costs of production is especially worthwhile in an environment of high prices.

We can observe a crucial major deficiency, not only in the Barone's original analysis, but in several contributions of his followers as well (see above). There is a clear lack of discussion on the interactions between the price mechanisms in the overall economy and the occurrence of innovation and imitations (see Külp 2017, pp. 56-57). We intend to overcome this deficiency by explicitly considering the forces of demand. More precisely, we depict in the right half of Figure 2 the well-known AS/AD-analysis. Given the normal AS- and AD-curves, the equilibrium price level P0 is achieved. This will enable a differential profit of size DP taking into account the shift in the old Barone's Curve B0 to its new position B1, where some sort of innovation boost has taken place. Presuming that a relatively high level of prices is responsible for this event and that the Schumpeterian entrepreneurs, as pioneers, are willing to implement innovations, the income distribution will change and will improve profits, while it will be detrimental to wages. As shown in Figure 2, DP will increase at a constant denominator (Y times P). In this case, the new Barone's Curve B1 is not associated with an expansion of the production level. The new marginal supplier is still the old one. As a result, the imitating competitors, attracted into the market by the increase in profits, will provoke a boost in the imitations in such a way that B2 will become the new relevant Barone's Curve. Consequently, the output will expand (now AS' replaces AS) while the price level will fall from P0 to P1. At the same time, the income distribution will now change in favor of labor income, but it will become detrimental to capital income.

Up to now, we have not questioned the possible behavior of the AD-curve. Its behavior essentially depends, as we know, on the price elasticity of demand. If the AD curve is inelastic and thus steep, as it is the case for AD', an increase in total supply will lead to a pronounced fall in the price level and a concomitant mild expansion of output (in comparison with the AD scenario). Conversely, a quite elastic curve of total demand, as it is the case for AD'', will provoke a much less relevant price decrease, while we would register a quite considerable output expansion (GDP). When comparing these two cases, it seems clear that only the latter event will somehow allow the Schumpeterian entrepreneurs to keep on frequenting this market/economy and will hopefully avoid them to focus on other markets/economies.

Hence, this raises the question: which are the most important determinants for the price elasticity of the overall demand curve AD? We could certainly tackle this subject from different points of view; either from a macroeconomic perspective, that is, within the framework of a traditional AS/AD-model, or making use of structural economics/microeconomics/international economics (i.e., as in the 2004 Samuelson model).

In the following section, we discuss about the shape and properties of the AD function. In equations (1) to (4), we find the definitions of the IS and of the LM curve (simple and modified) as well as of

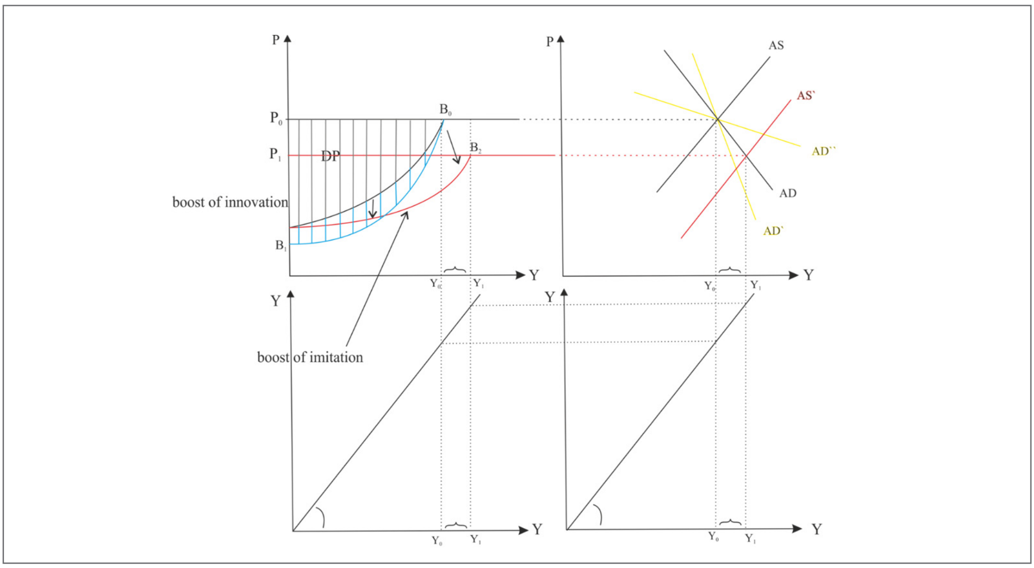

Figure 2. Innovations, imitations, output (Y) and the price level (P).
Sources: Külp, 2017; own compilation.

the money supply in real terms.

For the IS curve, $\alpha \equiv 1/(1-c+ct)$, $A_\alpha \equiv$ domestic autonomous absorption. Equation 2 is the LM curve, whereas in Equation 3, $M^n \equiv$ nominal money supply. Equation 4 shows the modified LM curve.

When plugging the value of the LM curve into the definition of the IS curve, by using Equations (5) to (8), we get Equations (9) and (10), which are two equivalent expressions of the AD function.

In Equations (11) and (12), we offer two alternatives, but still, we make equivalent calculations for the price elasticity of the AD curve. On the upcoming two pages, we discuss some of the key determinants of the mentioned price elasticity.

$$Y = \alpha(A_\alpha - hi) \tag{1}$$

$$i = \frac{1}{j}(kY - M) \tag{2}$$

$$M = \frac{M^n}{P} \tag{3}$$

$$i = \frac{1}{j}\left(kY - \frac{M^n}{P}\right) \tag{4}$$

(4) in (1)

$$Y = \alpha\left[A_\alpha - \frac{h}{j}\left(kY - \frac{M^n}{P}\right)\right] \tag{5}$$

$$\left[1 + \alpha\frac{h}{j}k\right]Y = \alpha\left[A_\alpha + \frac{h}{j}\frac{M^n}{P}\right] \tag{6}$$

$$\left[\frac{j+\alpha hk}{j}\right]Y = \alpha\left[A_\alpha + \frac{h}{j}\frac{M^n}{P}\right] \tag{7}$$

$$\beta = \frac{j\alpha}{j+\alpha hk} \tag{8}$$

$$Y = \beta\left(A_\alpha + \frac{h}{j}\frac{M^n}{P}\right) \tag{9}$$

$$P = \frac{\beta h}{j}\frac{M^n}{(Y-\beta A_\alpha)} \tag{10}$$

$$\varepsilon_{Y,P} = \frac{\partial Y}{\partial P}\frac{P}{Y} = -\frac{\beta h}{j}\frac{M^n}{P^2}\frac{P}{Y} = -\frac{\beta h}{j}\frac{M^n}{PY} < 0 \tag{11}$$

$$\varepsilon_{Y,P} = \frac{\partial Y}{\partial P}\frac{P}{Y} = -\frac{\alpha h}{(j+\alpha hk)}\frac{M^n}{PY} < 0 \tag{12}$$

In general terms, we can conclude that the higher the price elasticity of demand, the flatter the AD curve. However, in order to determine the factors on which depends, we consider some specific cases, no matter whether the slope of the AD curve is steep or flat:

- The higher the interest rate sensitivity of money demand (j), the steeper the AD-curve, as $|\varepsilon_{Y,P}|$ will become smaller and its curvature will tend to be more inelastic, as a result:

$$\frac{\partial \varepsilon_{Y,P}}{\partial j} = \frac{\alpha h}{(j+\alpha hk)^2}\frac{M^n}{PY} > 0$$

- the greater the cash holdings coefficient (k), the steeper the AD-curve. $|\varepsilon_{Y,P}|$ will become smaller and hence its curvature tends to be more inelastic:

$$\frac{\partial \varepsilon_{Y,P}}{\partial k} = \frac{\alpha^2 h^2}{(j+\alpha hk)^2}\frac{M^n}{PY} > 0$$

- the higher the interest rate sensitivity of private investment demand (h), the flatter the AD-curve, as $|\varepsilon_{Y,P}|$ will become larger and hence its curvature will tend to be more elastic:

$$\frac{\partial \varepsilon_{Y,P}}{\partial h} = -\frac{\alpha j}{(j+\alpha hk)^2}\frac{M^n}{PY} < 0$$

- the larger the size of the multiplier (α), the flatter the AD-curve, as $|\varepsilon_{Y,P}|$ will become larger and hence its curvature will tend to be more elastic:

$$\frac{\partial \varepsilon_{Y,P}}{\partial \alpha} = -\frac{hj}{(j+\alpha hk)^2}\frac{M^n}{PY} < 0$$

- the higher the aggregate demand is, the greater the domestic autonomous absorption will be and the larger the real money supply will be as well:

$\frac{\partial Y}{\partial A_\alpha} = \beta > 0$ and $\frac{\partial Y}{\partial M^n/P} = \frac{h}{j}\beta > 0 \rightarrow$ shift of the AD-curve to the right

Which is the core of the detailed considerations for elasticities that we have introduced in our new comparative-static analysis of the Barone's Curve above presented? Any boost in imitation (expansion of supply) will lower the price level even more, and as a result, innovation will be less likely to occur and the curvature of the AD-curve will be more inelastic. If, on the contrary, this event is more likely to happen, the interest rate sensitivity of money demand will be higher (falling into a liquidity trap!). Besides, the cash-holding coefficient (meaning that the income velocity of money is low) will be greater. Finally, the interest rate sensitivity of private investment demand (known as the investment trap) and the income multiplier will be lower, while the marginal propensity to save and to import will be higher.

Either from a structural-economics perspective or from a microeconomics point of view, we should be interested not only in the composition of GDP, but also in the respective price elasticities of its major components. For example, we can observe lower price elasticities in the primary sector (foodstuff), along with other sectors such as services, rentals, medical care, and local services. We find comparatively high elasticities in the sectors of tradable semi luxury foodstuff (as alcohol or drugs), electricity usage, and donations for charity and in the sector of manufactured goods (as automobiles). Presumably, the production of cars and car oriented upstream services composes a sector that particularly incorporates innovation-close technologies (Nicholson 1992, p. 211).

From an international-economics point of view, Samuelson (2004) argues that some countries such as China (especially vis-à-vis the USA) will experience a relative depreciation on its exports and a concomitant loss in terms of trade. This happens whenever the country concentrates its efforts on the raise of productivity in the sector of its own export goods; a sector which, by now, exhibits rather low price elasticities. Therefore, the goods and services that show comparatively high price elasticities of demand are less likely to suffer from price slumps when an expansion of supply takes place (See Figure 2 for further details). These goods or services will rather experience a moderate price decline, ceteris paribus, and thus will be attractive for innovators in the future.

4. Explaining the Dynamics of Prices Extending the Original Barone's Curve Framework

In the following discourse, we present a dynamic model, focused from a supply side point of view with regards to the Barone's Curve, in which demand is considered exogenous. As a result, we achieve a non-homogeneous differential equation of the second order, which makes sense to solve. Meanwhile, we also consider the capacities got via innovations and/or imitations. In contrast to Meyer (1989, pp. 309-311), we renounce to distinguish between the two notions of selection and invention. However, we will take up, the issue of inventions in the empirical part of the paper, where patents figure prominently.

In Figure 3, DE stands for aggregate demand, while total supply SU is fragmented into two segments: one representing the capacities installed by innovators and the other installed by imitators. The associated part of the supply curve is then flat for innovators and comparatively steep for imitators. Speaking technically, we aggregate the individual marginal cost curves of innovators or imitators to group-specific limbs of total supply. Altogether, we achieve a kinked supply curve.

$$IM_t = \alpha\,(IN_t - IN_{t-1}) + \beta \tag{13}$$

Imitators (IM) will be oriented toward the behavior of innovators (IN). In analogy to Samuelson's acceleration principle, we here assume that it is the change in the activity of innovators that induces the occurrence of imitations. However, there exists a second, autonomous motive for the activity of imitation symbolized by the parameter (β).

$$IN_t = \gamma\, P_{t-1} \tag{14}$$

Innovators will react positively, although most likely with some time lag with respect to high prices or likewise high revenues per unit of production (this matches best with the views of both Barone and his follower Helmstädter). Under these circumstances, technical progress, which lowers costs per unit of production, will result in attractive difference profits. This terminology reveals another important aspect: Barone's concept does not cover every aspect of the Schumpeterian-pioneering entrepreneur. It is obvious that Barone was not interested in product innovations, but only in the phenomenon of organizational or technological innovations in the sphere of production (and not of final product sales).

$$P_t = P_{t-1} + \{DE - (IN_t + IM_t)\} \tag{15}$$

According to Equation (15), the price level will always change, whenever there is disequilibrium between supply (SU) and demand (DE).

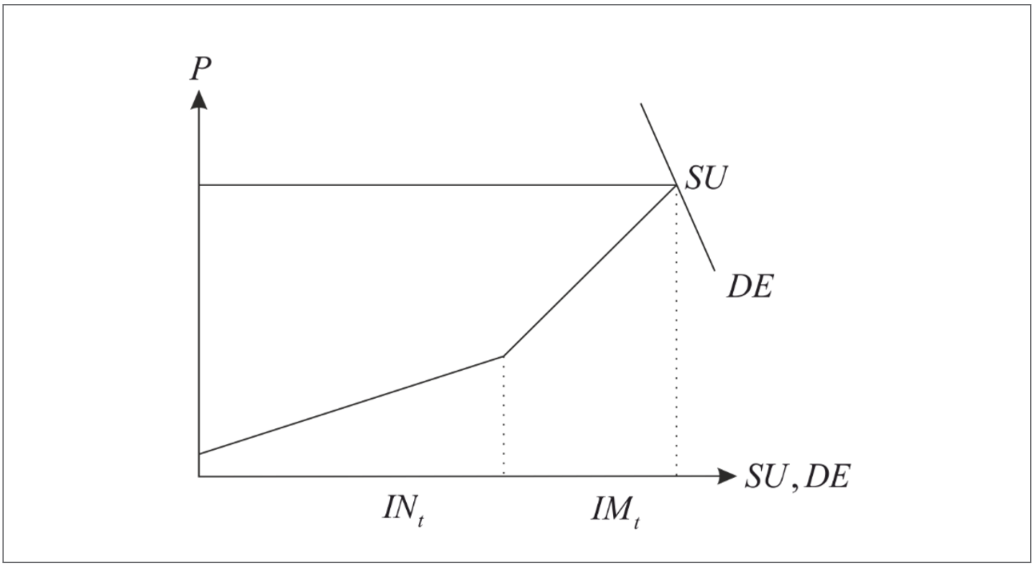

Figure 3. Innovations, imitations, output (Y) and the price level (P).

$$SU = IN_t + IM_t \tag{16}$$

The relevant supply is attributable either to innovators or to imitators:

$$DE = \overline{DE} \tag{17}$$

Demand is taken to be exogenous.

$$SU = DE \tag{18}$$

In principle, we postulate the rule of market equilibrium, according to Equation (18). By consolidating Equations (13) to (18), we obtain the following inhomogeneous difference equation of the second order:

$$P_t = P_{t-1} + \{\overline{DE} - \gamma P_{t-1} - \alpha\gamma P_{t-1} - \alpha\gamma P_{t-2} - \beta\} \tag{19}$$

We can interpret the price cycle caused by innovations and imitations (and their mutual interdependence) in the following way: when prices are high, large innovations will take place (despite the time lag), and the latter will attract imitators. Whenever supply rises sufficiently in comparison to demand, a price decline follows necessarily. This development tends to dampen innovations, and, therefore imitations will smooth as well. Once this effect has proven be significant enough for the change in demand, price increases are to be expected. This event will in turn encourage innovations again and will trigger the full price cycle over again.

From above, we have:

$$P_t = \overline{DE} + P_{t-1}(1 - \gamma - \alpha\gamma) - \alpha\gamma\, P_{t-2} - \beta \tag{20}$$

$$\overline{DE} - \beta = P_t - P_{t-1}(1 - \gamma - \alpha\gamma) + \alpha\gamma\, P_{t-2} \tag{21}$$

In the mathematical annex, we solve our nonhomogeneous differential equation problem of the second order in two steps. Here, in the core text, we present our main results in a graphical way (see Figure 4)

In Figure 4, we can observe two branches of the oscillation line as well as the vertical convergence line, both as a function of the parameters alpha and gamma. As a result, we get six cases for the develop-

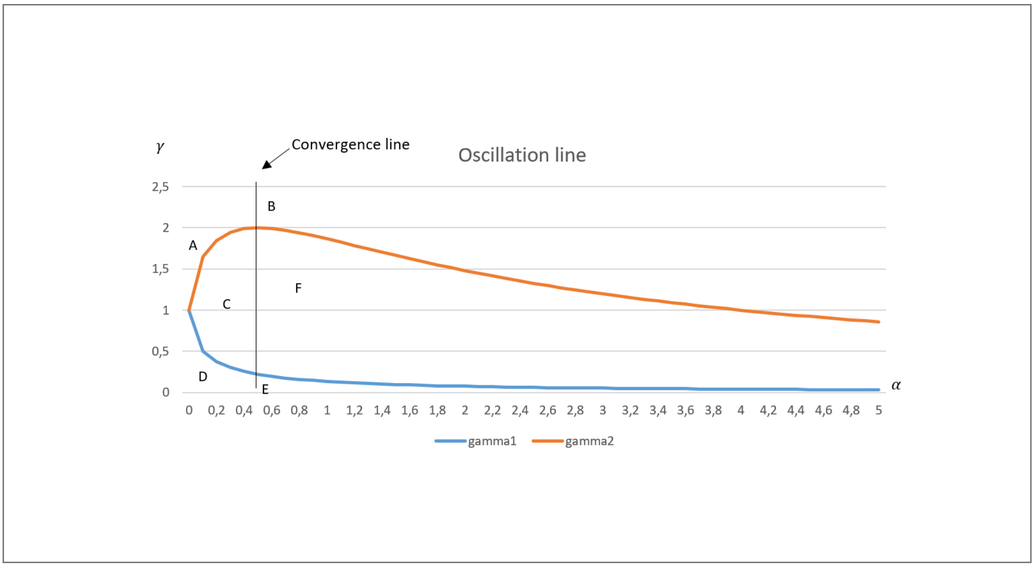

Figure 4. Oscillation and convergence lines in the dynamic Barone's Curve Modell.

Table 2. Convergence vs. steadiness of price development in the dynamic Barone's Curve Model (case analysis).

Case A	Continuous and convergent
Case B	Continuous and divergent
Case C	Discontinuous and convergent
Case D	Continuous and convergent
Case E	Continuous and divergent
Case F	Discontinuous and divergent

ment of prices, which are systematically distinguished in Table 2. We can highlight that the upper part of the oscillation line has a local maximum exactly at a value of α as 0.5; is the point where the convergence line is located.

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Synthesizing all the six cases from Table 2, and building four sub-groups, leads to the system of events for the development of prices depicted in Figure 5. In principle, there may exist a fifth group, which represents the case for constant oscillations, which is a well-known variety of Samuelson's model

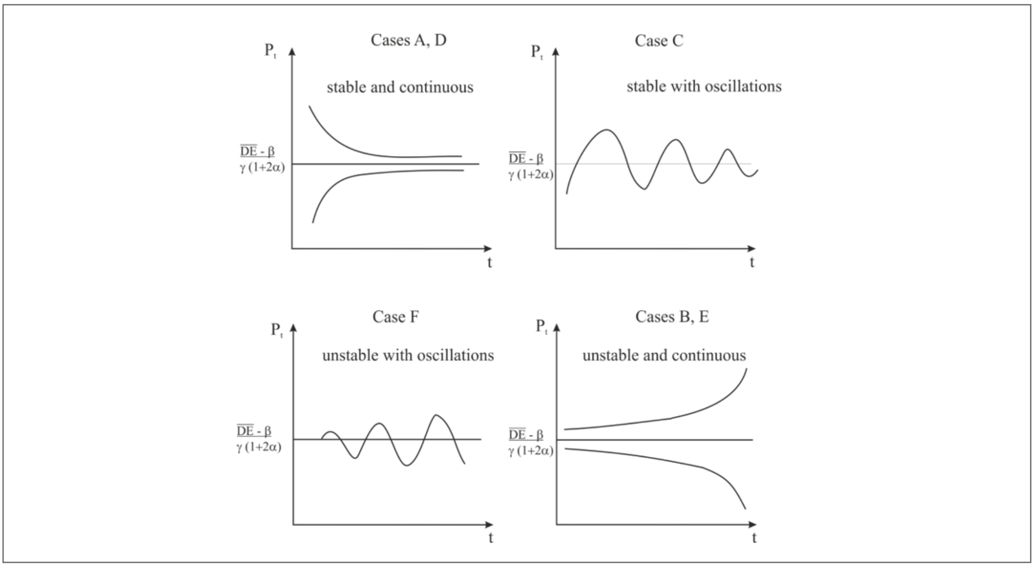


Figure 5. Stable/unstable and steady/oscillating development of prices in the Barone's Curve Model.

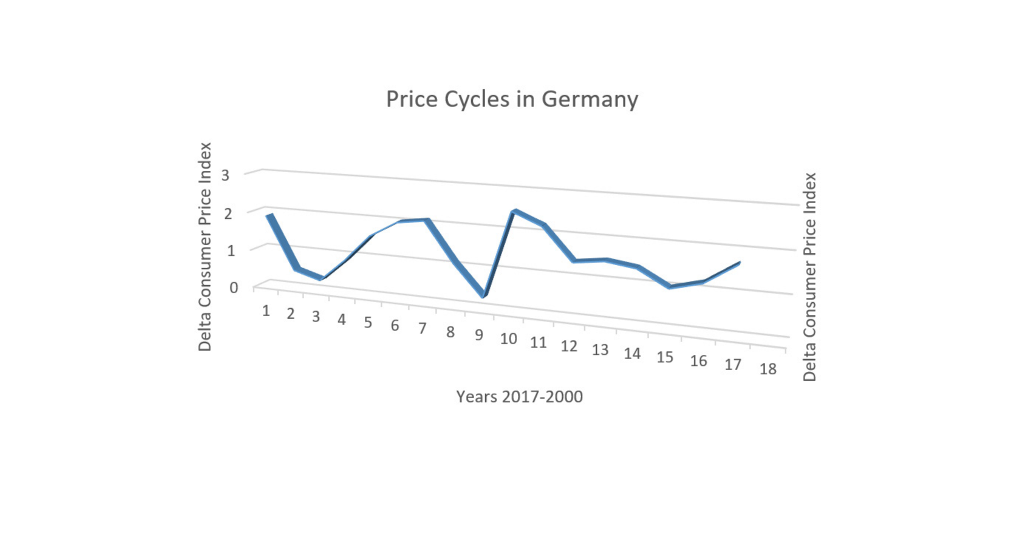
(1939). This scenario would set a framework with recurring cycles along with a constant amplitude and a fixed frequency

5. Earlier and Recent Empirical Research Related to the Barone's Curve

Figure 6 shows in a graphic way the changes (first differences) in the price index of consumer expenditures (top part) as well as in the GDP deflator index (bottom part). The numbers are derived from the data for Germany for the time-period between 2000 and 2017. We can observe that the absolute values of the price level are strongly influenced by the trend factor and they exhibit a unit root, meaning that they are integrated of order 1. As a result, we take up the suggestions of the theory of Co-integration (Stock & Watson 2012, pp. 684–701) and we use only first differences for the empirical study (if they are not available, we use differences of a higher order). We highlight that these newly built time series are fixed. In Figure 6 we can observe a clear cycle pattern as well, as it was postulated in the dynamic modeling (Section 4), especially in the cases above presented under the names of B and C.Helmstädter (1986a und 1986b) suggested that the distribution function of revenue margins should be considered, for example in the sector of manufactured goods. According to him, we should expect the Schumpeterian pioneers to exhibit above-average revenue margins while imitators will most likely achieve below-average margins only.

If the distribution function of revenue margins changes over time, in a way that the share of firms with zero profits increases and the distribution function becomes flat where low profits are earned (Helmstädter 1986a, p. 30), then we may presume that markets are less profitable than how they were used to be. In this scenario, a less favorable slope of profits goes along with the deterioration in the revenue-cost relationship. The latter phenomenon can be a strong indicator of an extremely low intensity of innovation activity. However, this conclusion may be incorrect. Why? The erosion of profits may simply indicate a very successful activity of the imitators who contribute to reduce the slope of profits (see Blümle 1989, pp 26–30).

As the saying goes, "no smoke without a preceding fire"; we may suspect that innovations were implemented first and imitators were pulled into the market later



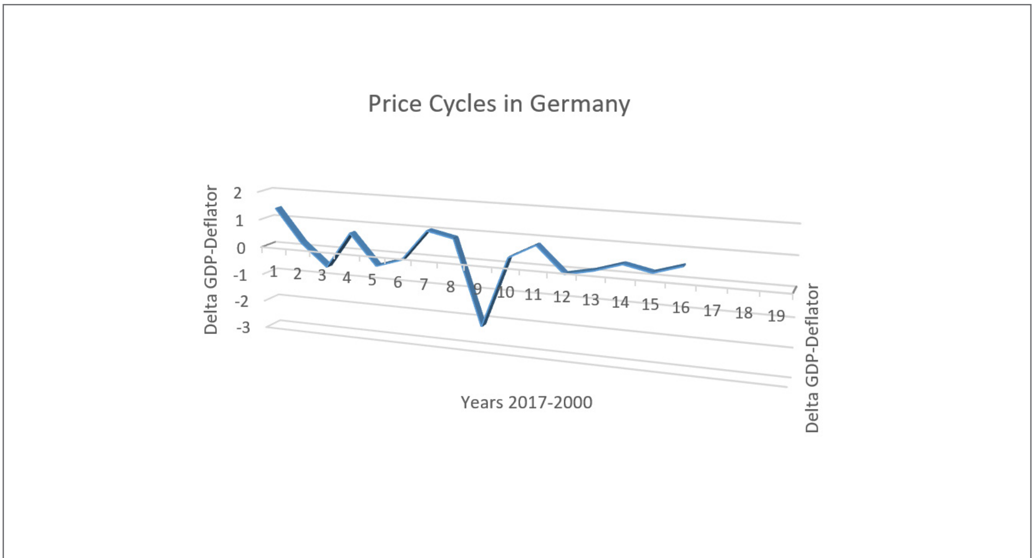


Figure 6. Delta Consumer Price Index and Delta GDP-Deflator in Germany between 2000 and 2017.
Sources: Statista 2018; own calculations.

If we try to identify viable indicators for the activity of innovations and imitations, we can use, for example, the number of registered patents and the number of registered/marketed licenses, respectively. Besides this, it seems worthwhile to search statistics on license fees. In practice, however, it is hard to find any relevant information about such fees. In principle, expenditures on research and development might serve as the information source, but they imply a great problem for identification, as these expenditures usually include both imitations as well as innovations.

A direct and simple approach to explain the record of patents is the well-known random walk model (Spahn 2001, p. 9): inventors and their patent record activities follow a stochastic pattern of error and trial, which is not steered by the so-called fundamental (economic) variables. However, inventions are not yet innovations. For the an effective innovation to take place, there is a need for an entrepreneur who, in the view of Israel M. Kirzner (1978), is not interested in the possession of resources or specific goods or services, but who just wants to take advantage of the possibilities to earn some money. The Barone's Curve fits these interests well as it offers the chance to raise the benefits gains per unit of output (Kirzner 1978) by reducing the unit costs of production.

In the following discourse, we execute a linear regression (see Figure 7), testing econometrically equation (22):

$$PAT_t = \beta_0 + \beta_1 PAT_{t-1} + \varepsilon_t \quad (22)$$

According to Equation 22, we assume that the time series of patent records exhibits a stochastic trend and is hence nonstationary (Stock & Watson 2012, pp. 589-90).

One-period delayed domestic patent records and a stochastic term explain domestic patent records in Germany, as it is commonly used when applying the random walk approach. The estimated coefficient β1 is statistically significant at a 99% level ($p < 0.01$), a result, which confirms the assumption, we departed from a non-stationarity of the times series of patent records.

6. Policy implications and conclusions

If volatile price movements are explained by the nature of innovation and imitation cycles, we question whether macroeconomic policy in general and fiscal policy in particular, are well advised when they pursue a rigorous stabilization policy. More precisely, when such policies aim such a rigid goal as a constant (and low) inflation rate.

Earlier empirical studies covered the period before the "great moderation" and reported that higher (lower) rates of inflation were accompanied by a stronger (weaker) volatility of price movements (Sell 1988, pp. 390-392 and Sell 1990, pp. 60-61). In a more recent past, there has hardly been conducted a single study

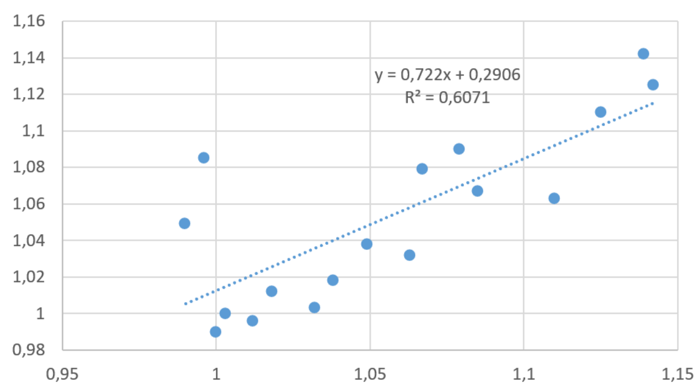


Figure 7. A random-walk-model of patent registrations in Germany (index values with 2010 = 1; 2000–2017).
Sources: Statista 2018; own calculations.

on this subject, presumably because higher (lower) inflation rates have become rarer (more frequent).

We conclude that we should also be careful when using competition policy to determine which product markets to subject to price reductions or price increases (Helmstädter 1986a, p. 24). Such an attempt would stand in sharp contrast to the concept of the Barone's Curve. Competition policy should rather be obliged to guarantee the "freedom of competition" (Hoppmann 1967) and should relinquish to evaluate the results of the market competition. Otherwise, it would ultimately fall into the so-called "v. Hayek-trap". Competition policy would assume a perfect knowledge of the interaction process between innovations and imitations, which is definitely not at its disposal. Therefore, the aim of public economic policy cannot be to discovery of new fields of action in the promotion of research and new technologies (Helmstädter 1986a, p. 32).

The Barone's Curve is of an unbeaten actuality and relevance. This statement is also valid in this time of robotization and digitalization. It was our aim to further develop the static and the dynamic properties and implications of the Barone's Curve Model. Basically, we have added three elements to the existing research: one is the addition of the total demand variable to the supply-oriented that characterizes the Barone's Curve Model, which enables us to discuss the macroeconomic price formation process. This is an indispensable aspect when it comes to explaining the boost of either innovation or imitation. A second key result of our analysis consists of the detection of price cycles as an outcome of the dynamic Barone's Curve modeling. Such cycles presents a challenge to modern macroeconomic stabilization policy. Thirdly and finally, we empirically tested the determinants of patent records, certainly considered as a sort of precursor for any type of innovation. The random walk model was found to fit best to the set of actual data from Germany (2000¬–2017).

References

Barone, E. (1935). *An Introduction to Theoretical Economics*. Dümmlers.

Blümle, G. (1989). Economic growth and the business cycle - a schumpeterian model of economic development. In H. J. Ramser, & H. Riese (Eds.), *Contributions to applied economic research* (pp. 13–37). Springer.

Bradley, M. E., & Mosca, M. (2012). Enrico Barone's "Ministry of Production": Content and context. *The European Journal of the History of Economic Thought*, *21*(4), 664–698.

Deutsches Patent und Markenamt (2019). The number of patent registrations in Germany between 2000 and 2017. Retrieved from https://de.statista.com/statistik/2203/umfrage/entwicklung-der-anzahl on January 2, 2019.

Helmstädter, E. (1986a). New approaches to dynamic competition. In: Research institute for economic order and competition (Ed.), *A new orientation in the protection of competition,* 25 years of FIW (pp. 23–33). Heymann.

Helmstädter, E. (1986b). Dynamic competition, economic growth and employment. In G. Bombach, B. Gahlen, & A. E. Ott (Eds.), *Technological change – analysis and facts* (pp. 67–82). Mohr.

Hoppmann, E. (1967). Competition as a standard for competition policy. *ORDO: Jahrbuch für die Ordnung von Wirtschaft und Gesellschaft*, *18*, 77–94.

Iwai, K. (1984). Schumpeterian dynamics: An evolutionary model of innovation and imitation. *Journal of Economic Behavior & Organization*, *5*(2), 159–190.

Iwai, K. (1984). Schumpeterian dynamics, part II: Technological progress, firm growth and 'economic selection'. *Journal of Economic Behavior & Organization*, *5*(3-4), 321–351.

Kirzner, I. M. (1978). *Competition and entrepreneurship (Vol. 14).* Mohr.

Külp, B. (2017). *Functions and dysfunctions of profits.* Mimeo.

Meyer, B. (1989), An evolutionary interpretation of the Barone's curve, in Gahlen, B., Meyer, B. & Schumann, J. (Eds.), *Economic growth, structural change and dynamic competition* (pp. 307–319). Springer.

Nicholson, W., & Snyder, C. M. (2012). Microeconomic theory: basic principles and extensions. Nelson Education.

Samuelson, P. A. (1939). Interactions between the multiplier analysis and the principle of acceleration. *The Review of Economics and Statistics*, *21*(2), 75–78.

Samuelson, P. A. (2004). Where Ricardo and Mill rebut and confirm arguments of mainstream economists supporting globalization. *Journal of Economic Perspectives, 18*(3), 135–146.

Sell, F. L. (1988). *Monetary and exchange rate policy in emerging economies. The example of the ASEAN* *countries*. Duncker & Humblot.

Sell, F. L. (1990). Inertial inflation and heterodox stabilization policy. Experiences from Argentina, Brazil and Israel. *Kredit und Kapital*, *23*(1), 60–85.

Sell, F. L. (2019). Innovations, imitations and differential profits: The Barone's curve in the context of economic development. *Working Papers in economics No. 1. Bundeswehr* University Munich.

Spahn, H. P., & Bliss, C. (2001). Interest rate parity, the exchange rate and monetary policy. *Jahrbuch für Wirtschaftswissenschaften, 52*(1), 1–24.

Statista. (2018). The consumer price index in Germany, 1991 through 2017. *Statista für Universitäten, Bibliotheken und Studenten. Das Recherche - und Analysetool*, [Data set]. Retrieved from https://de.statista.com/statistik/daten/studie/2550/umfrage/entwicklung-des-verbraucherpreisindex/ on January 10, 2019.

Stock, J. H., & Watson, M. M. (2012). *Introduction to Econometrics (Vol. 3).* Pearson.

Appendix

First, we begin with the stationary (inhomogeneous part of the) solution.

Inhomogeneous Solution:

$$DE - \beta = P - P(1-\gamma-\alpha\gamma) + \alpha\gamma P$$

$$\overline{DE} - \beta = P - P + P\gamma + \alpha\gamma P + \alpha\gamma P$$

$$\overline{DE} - \beta = P\,(\gamma + 2\alpha\gamma)$$

$$\frac{\overline{DE}-\beta}{(\gamma+2\alpha\gamma)} = P$$

Hence, the price effects in the stationary (inhomogeneous) solution will be higher when

- the expansion of exogenous demand is stronger,
- the size of autonomous imitations is smaller,
- the reaction of innovators with regard to the previously achieved price level is more insignificant,
- the impact of changes in innovations on the behavior of imitators is lower.

The next step is identifying the dynamic, homogeneous part of the solution:

Homogeneous Solution:

$$0 = H^2 - (1-\gamma-\alpha\gamma)H + \alpha\gamma$$

$$H_{1/2} = \frac{(1-\gamma-\alpha\gamma)}{2} \pm \sqrt{\left[\frac{(1-\gamma-\alpha\gamma)}{2}\right]^2 - \alpha\gamma}$$

$$H_{1/2} = \frac{(1-\gamma-\alpha\gamma)}{2} \pm \sqrt{\frac{(1-\gamma-\alpha\gamma)^2}{4} - \alpha\gamma}$$

$$H_{1/2} = \frac{(1-\gamma-\alpha\gamma)}{2} \pm \frac{\sqrt{(1-\gamma-\alpha\gamma)^2 - 4\alpha\gamma}}{2}$$

$$H_{1/2} = \frac{(1-\gamma-\alpha\gamma) \pm \sqrt{(1-\gamma-\alpha\gamma)^2 - 4\alpha\gamma}}{2}$$

Henceforth, we test the fulfillment of the Schur criteria:

Schur–Criteria (Examination of Convergence):

$$1 - (1-\gamma-\alpha\gamma) + \alpha\gamma > 0$$

$$1 + (1-\gamma-\alpha\gamma) + \alpha\gamma > 0$$

$$1 - \alpha\gamma > 0$$

$$\gamma + \alpha\gamma + \alpha\gamma > 0$$

$$\left.\begin{array}{l} 2-\gamma > 0 \\ \quad 2 > \gamma \\ 1-\alpha\gamma > 0 \\ 1 > \alpha\gamma \end{array}\right\} \quad \left.\begin{array}{l} 2 > \gamma \\ \\ \\ 2 > 2\alpha\gamma \end{array}\right\} \quad \alpha < \tfrac{1}{2}$$

Convergence line

Value of the Discriminant (Examination of cycles):

$$\sqrt{\underbrace{(1-\gamma-\alpha\gamma)^2 - 4\alpha\gamma}_{\Delta<0}}$$

$$\Delta := [1-\gamma(1+\alpha)]^2 - 4\alpha\gamma$$

$$= 1 - 2\gamma(1+\alpha) + [\gamma(1+\alpha)]^2 - 4\alpha\gamma$$

$$= \gamma^2(1+\alpha)^2 - 2(1+3\alpha)\gamma + 1$$

The Discriminant is for each $\alpha \neq -1$ a quadratic function in γ and can be factorized via the zero points. The zero points are:

$$\gamma_{1,2} = \frac{(2+6\alpha) \pm \sqrt{(2+6\alpha)^2 - 4(1+\alpha)^2}}{2(1+\alpha)^2}$$

$$= \frac{(1+3\alpha) \pm \sqrt{(1+3\alpha)^2 - (1+\alpha)^2}}{(1+\alpha)^2} = \frac{(1+3\alpha) \pm 2\sqrt{\alpha+2\alpha^2}}{(1+\alpha)^2}$$

Remark: It can be easily shown that for $\alpha \geq 0$, it holds $\gamma_{1,2} > 0$ (Restriction: $0<\gamma_1\leq\gamma_2$). From here we may deduct for Δ, the subsequent factorization:

$$\Delta = \gamma^2(1+\alpha)^2 - 2(1+3\alpha)\gamma + 1$$

$$= (1+\alpha)^2(\gamma-\gamma_1)(\gamma-\gamma_2).$$

Hence, we achieve the following two cases:

In case 1 with $\gamma_1<\gamma<\gamma_2$ we have $\Delta<0$ (and hence complex solutions); otherwise (case 2), we have $\Delta\geq 0$ (real solutions).

The points $(\alpha;\gamma)$ with $\Delta=0$ decribe the oscillation line depicted in Figure 4. The oscillation line sort of designs the crossing space between real and complex solutions of the difference equation.

(a) Condition for conjugated complex solutions (oscillation line): the areas C and F (cf. Table 2) contain those points associated with case 1, with a negative discriminant Δ, which follows from the factorization.

(b) Roots are different, but real numbers $H_1 \neq H_2$. For any $\Delta>0$, the inhomogeneous difference equation has real solutions. This is the case for all pairs $(\alpha;\gamma)$ with $\alpha\geq 0$ and $0<\gamma<\gamma_1\,(\alpha)$ (cf. Table 2, areas D and E).

Or it applies to the case $\gamma > \gamma_2$ (α) (cf. Table 2, areas A and B).

(c) Simple case $\Delta = 0 => H_1 = H_2 = \frac{(1-\gamma-\alpha\gamma)}{2}$

In the following Table 1, we demonstrate a simulation of both the oscillation lines. Given the existence of two oscillation lines, we achieve in total six cases to be distinguished (A through F). The latter ones we present in Table 2 (main text).

Table 1. Numerical simulation of solutions for γ_1, γ_2 and α.

alpha	gamma1	gamma2
0	1	1
0.1	0.50180139	1.64695894
0.2	0.37618019	1.84604203
0.3	0.30435465	1.94416606
0.4	0.25660394	1.98829402
0.5	0.22222222	2
0.6	0.19616209	1.99133791
0.7	0.17567603	1.96965269
0.8	0.15912314	1.93964229
0.9	0.14545663	1.90440487
1	0.1339746	1.8660254
1.1	0.12418762	1.82592576
1.2	0.11574339	1.78508305
1.3	0.10838156	1.74417042
1.4	0.10190538	1.70365018

Table 1. Numerical simulation of solutions for γ_1, γ_2 and α (Continued).

alpha	gamma1	gamma2
1.5	0.09616328	1.66383672
1.6	0.0910366	1.62493973
1.7	0.08643104	1.58709434
1.8	0.08227069	1.55038238
1.9	0.07849375	1.51484751
2	0.07504941	1.48050615
2.1	0.07189546	1.44735532
2.2	0.06899656	1.41537844
2.3	0.06632292	1.38454944
2.4	0.0638492	1.35483592
2.5	0.06155374	1.32620136
2.6	0.0594179	1.29860679
2.7	0.05742558	1.27201197
2.8	0.05556274	1.24637632
2.9	0.05381714	1.22165952
3	0.05217804	1.19782196
3.1	0.05063596	1.17482507
3.2	0.04918254	1.15263152
3.3	0.04781032	1.13120537
3.4	0.04651268	1.11051212
3.5	0.04528369	1.09051878
3.6	0.04411805	1.07119386

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Table 1. Numerical simulation of solutions for γ_1, γ_2 and α (Continued).

alpha	gamma1	gamma2
3.7	0.04301096	1.05250737
3.8	0.04195813	1.03443076
3.9	0.04095565	1.01693689
4	0.04	1
4.1	0.03908797	0.98359562
4.2	0.03821663	0.96770053
4.3	0.03738331	0.95229273
4.4	0.03658559	0.93735131
4.5	0.03582123	0.92285646
4.6	0.03508817	0.90878938
4.7	0.03438453	0.89513224
4.8	0.03370857	0.88186812
4.9	0.0330587	0.86898095
5	0.03243342	0.85645547