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# Testing the Perturbation Sensitivity of Abortion-Crime Regressions

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## ABSTRACT

The hypothesis that the legalisation of abortion contributed significantly to the reduction of crime in the United States in 1990s is one of the most prominent ideas from the recent “economics-made-fun” movement sparked by the book *Freakonomics*. This paper expands on the existing literature about the computational stability of abortion-crime regressions by testing the sensitivity of coefficients’ estimates to small amounts of data perturbation. In contrast to previous studies, we use a new data set on crime correlates for each of the US states, the original model specification and estimation methodology, and an improved data perturbation algorithm. We find that the coefficients’ estimates in abortion-crime regressions are not computationally stable and, therefore, are unreliable.

## KEY WORDS:

abortion, crime, computational stability, perturbation test

## JEL Classification:

C52, J13, K42

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## Introduction

In a famous and controversial paper, Donohue and Levitt (2001), hereafter DL, argued that the legalisation of abortion in the United States (US) in the 1970s may account for as much as one-half of the overall crime reduction in the US in the 1990s. According to the theory behind this result, increased availability of abortion led to fewer unwanted children, who are more likely to become criminals when they reach adulthood. This hypothesis has become one of the most widely discussed ideas from Levitt and Dubner’s (2005) *Freakonomics*, which was enormously popular among the general public.

DL’s empirical analysis was criticised for various reasons by Joyce (2004; 2009), Lott and Whitley (2007), Foote and Goetz (2008), Moody and Marvell (2010) and others. Donohue and Levitt (2004; 2008)

responded to some of these critiques; see also Joyce (2010) for a general overview of the debate about the impact of abortion on crime.

One recent criticism of DL’s abortion-crime regressions involves testing the computational stability of their results using numerical analysis and computational economics tools. In particular, Anderson and Wells (2008) have argued that the computational problem posed in DL is ill-conditioned because it is very sensitive to small amounts of perturbation in the data, and therefore, their regression results are not computationally stable. Anderson and Wells (2008) showed that the condition number,  $\kappa$ , which is an upper bound for the sensitivity of the least squares solution to data perturbations, takes a very large value ( $\kappa = 1,329,930$ ) for the basic regressions calculated by Donohue and Levitt (2001). Moreover, they calculated the bound on the relative error of the coefficients estimated by DL and found that it is too high to have any confidence in the estimated results. They concluded that there is not enough information in the data used by DL to mean-

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ingly estimate regression coefficients. Anderson and Wells (2008) also showed that DL's models suffer from collinearity and that the linear specification used in these models is problematic. Finally, they show that similar problems also affect the results in Foote and Goetz (2008).

McCullough (2010) complements the theoretical insights in Anderson and Wells (2008) using a visual diagnostic tool for computational stability Beaton et al. (1976). His results, obtained using simplified versions of DL's models, suggest that DL's regressions were too demanding for the data, and therefore, the estimated results are not numerically stable.

In this paper, we provide another test of perturbation sensitivity for DL's original abortion-crime models. We extend the analysis of McCullough (2010) by assuming exactly the same model specification and estimation methodology as used by DL. Additionally, we use a new data set for the study of crime regressions, collected recently by Moody and Marvell (2010), which allows for close replication of DL's basic results. Finally, we use a formal algorithm, proposed by Vinod (2009), for producing perturbed data sets.

The remainder of the paper is organised as follows. Section II presents an introduction to the methods of testing for computational stability using data perturbation, Section III introduces the data, Section IV offers empirical results, and Section V concludes.

## Testing computational stability using data perturbation methods

Testing the computational stability of regression coefficients using data perturbation was first proposed by Beaton et al. (1976). Their procedure consists of simulating a large number of perturbed data sets by adding uniformly distributed numbers from the range  $[-0.5, +0.499]$  to the values of relevant variables after the last published digit. They suggested various statistics for comparing the original (unperturbed) solution with perturbed data sets, e.g., the per cent of perturbed solutions that agree with the original solution to at least the first significant digit, comparing the mean or the median of the perturbed coefficients with the unperturbed coefficient, etc. Beaton et al. (1976) also introduced a visual device for testing perturbation sensitivity, which consisted of histograms for the perturbed regression coefficients with superimposed vertical lines represent-

ing the values of unperturbed coefficients. Such plots are called "BRB plots" by McCullough (2010).

A recent study by Vinod (2009) creates a large number  $J$  of perturbed data sets by making small changes to the data beyond the available (published) digits to estimate what proportion,  $\alpha$ , of  $J$  delivers conclusions (e.g., concerning statistical significance of estimated coefficients or policy conclusions of estimates) that are opposite to those in the original data set. Then, the given conclusion from the original study is said to be  $100(1-\alpha)\%$  perturbation robust. If  $\alpha$  is small, then the results of the original study can be considered to be computationally stable.

To produce perturbed data sets, Vinod (2009) proposed a simple algorithm to retain only the reliable digits of every perturbed variable and replace the trailing digits with suitably chosen random numbers; see Vinod (2009, pp. 207-208) for details. We follow his algorithm to produce perturbed data sets for our analysis.

## Data

We use data from a new comprehensive panel data set with crime statistics from each US state with several potential control variables gathered by Moody and Marvell (2010) for the purposes of general-to-specific crime equation modelling. We use the same variable set as the original basic models in DL (2001, Table IV, p. 404). The data set consists of annual state-level observations for the period 1985-1997; the number of observations is 663.

The dependent variables are logs of three types of crime (murder, violent crime and property crime) per 1,000 population. The main independent variable ("effective" abortion rate) and control variables are presented in Table 1. In all of our calculations, we also include the state and year indicator variables, which control for state-year effects. Foote and Goetz (2008) found that DL made a programming error and failed to include controls for state-year effects in their regressions. However, Moody and Marvell (2010) were able to replicate the basic results in DL even when state and year dummies are included. The number of reliable digits for each variable is taken from McCullough (2010), who provides a detailed justification for these choices. In our tests, we use Vinod's (2009) algorithm to perturb every continuous independent variable beyond the appropriate number of reliable digits listed in Table 1. We do not perturb the *shall* variable, which is discrete.

**Table 1.** Control variables and their numerical accuracy

Variable name	Definition	Reliable digits
Abortion	"effective" abortion rate per 1,000	3
Prisonpc	log(prisoners per capita), one year lagged	3
Policepc	log(police per capita), one year lagged	3
Unrate	unemployment rate	3
Rincpc	real income per capita	6
Prate	poverty rate	2
rwelpc15	real welfare payments per capita, 15 years lagged	5
shall	shall-issue concealed weapons law	-
beerpc	beer consumption per capita	2

Notes: The data sources are provided in Moody and Marvell (2010). The number of reliable digits is taken from Mccullough (2010).

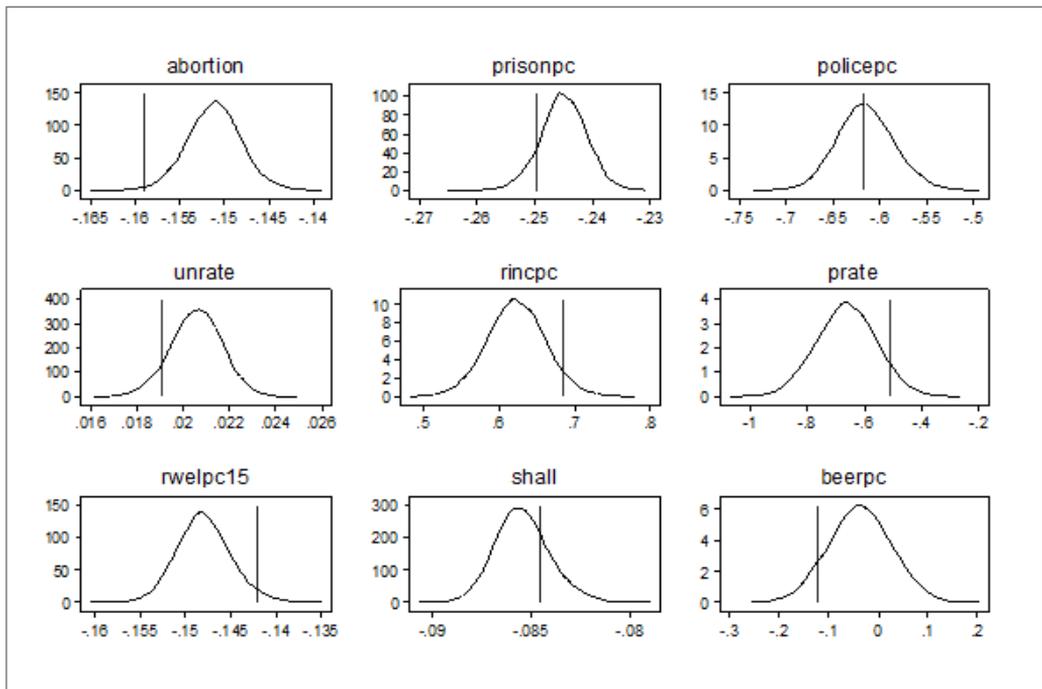
## Results

Similar to DL's original study, we use fixed-effects models to estimate the relationship between abortion and the three types of crime. Regressions are weighted by state population. The coefficient for the abortion variable is negative and statistically significant with  $p$ -values less than 0.01 for all three models. The results of our perturbation sensitivity tests are presented in Figures 1-3, which show the Monte Carlo distributions of perturbed regression coefficients for the main independent variables. The number of replications is 9,999. The vertical lines in Figures 1-3 represent the values of the original (unperturbed) coefficients. The expectation is, if the unperturbed solution is computationally stable for the data set under review, that the unperturbed solution is close to the centre of the distribution for the perturbed coefficients. However, for almost all independent variables from the abortion-crime regressions studied, the unperturbed coefficients are clearly significantly different from the means of the simulated distributions. The  $p$ -values for the tests to determine whether the unperturbed solution is statistically equal to the average of perturbed solutions are always less than 0.001.

Another simple test of computational stability using perturbed coefficients is to determine whether nearly

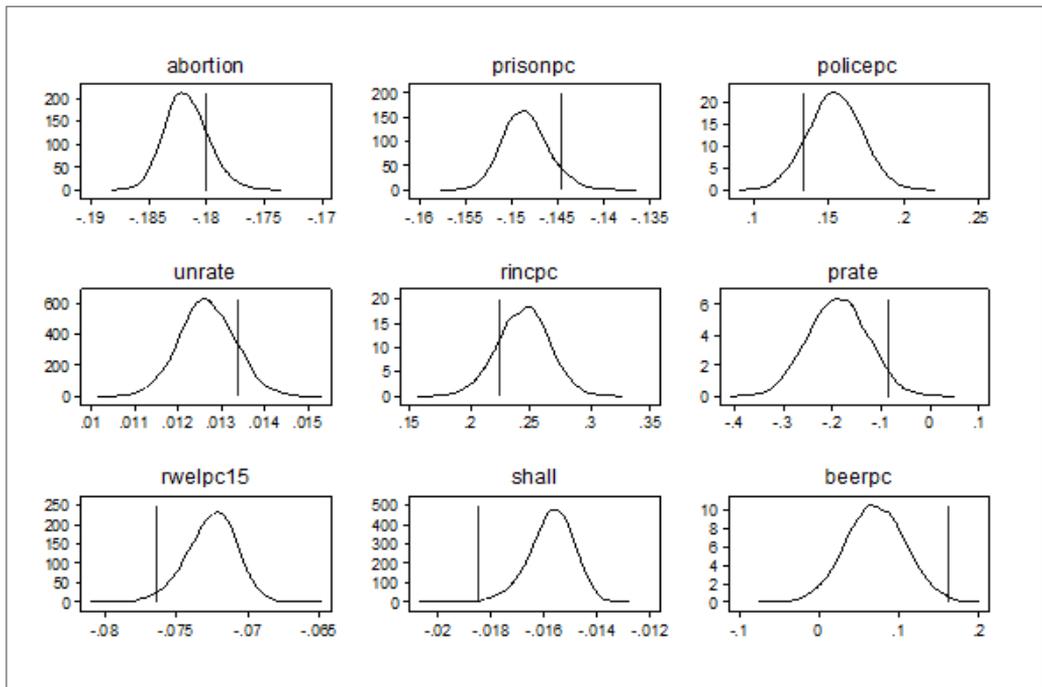
all of the perturbed solutions agree with the unperturbed solution to at least one significant digit (Beaton et al., 1976). The definition of agreement to one significant digit is as follows. If  $U$  is the unperturbed solution and  $P$  is a perturbed solution, then  $P$  is said to agree with  $U$  if  $P$  falls within the interval  $U \pm$  five units in the second significant digit of  $U$ .

In cases where regression results are computationally stable, all or almost all the perturbed coefficients should agree with their unperturbed counterparts to at least the first significant digit. Table 2 shows results of the test based on this idea. The perturbed coefficients for the abortion and prison variables always agree to a single significant digit with the unperturbed solution. However, this is not the case for other variables. In particular, for the *beerpc*, *prate* and *rincpc* variables, on average, only 15%, 21% and 69%, respectively, of perturbed coefficients agree with the unperturbed coefficients. In the case of the property crime equation, less than 1.5% of the simulated coefficients agree with the original coefficients for the *policepc* and *rwelpc15* variables, which is clearly a sign that, for our data set, the original (unperturbed) solution for the DL's abortion-crime regressions is not computationally stable.



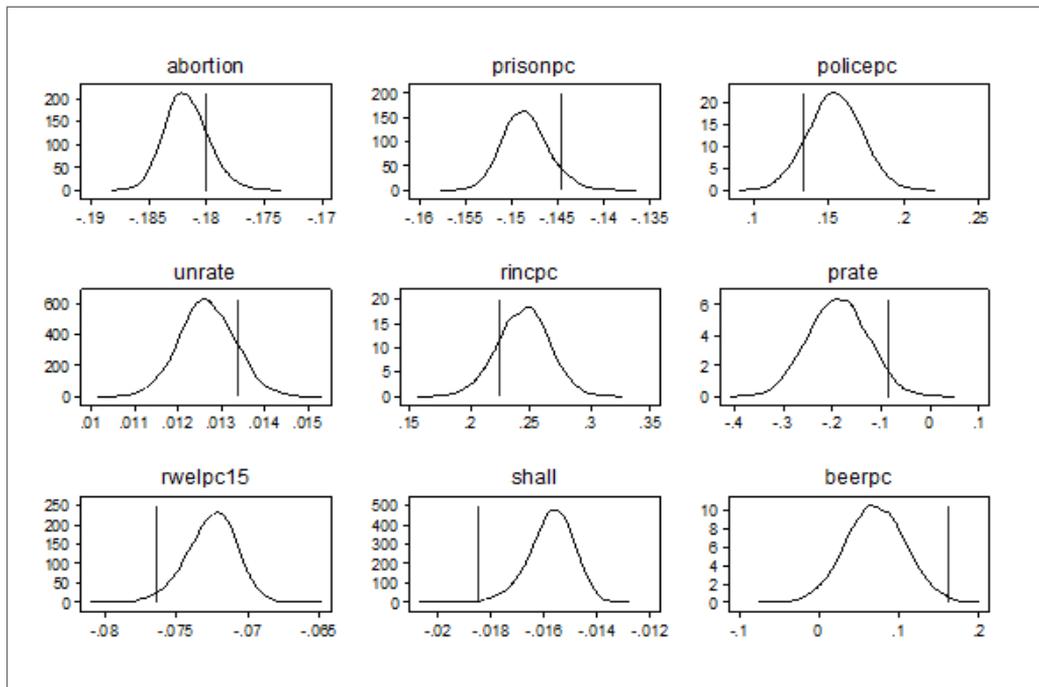
**Figure 1.** The Monte Carlo distribution of perturbed coefficients for the murder equation

Notes: The vertical lines represent the values of the unperturbed coefficients.



**Figure 2.** The Monte Carlo distribution of perturbed coefficients for the violent crime equation

Notes: The vertical lines represent the values of the unperturbed coefficients.



**Figure 3.** The Monte Carlo distribution of perturbed coefficients for the property crime equation

Notes: The vertical lines represent the values of the unperturbed coefficients.

**Table 2.** Per cent of perturbed coefficients that agree with the unperturbed solution to at least one significant digit

Variable	Murder	Violent crime	Property crime
abortion	100.00	100.00	100.00
prisonpc	100.00	100.00	100.00
policepc	89.54	94.92	0.79
unrate	99.94	100.00	100.00
rincpc	37.01	91.39	79.72
prate	13.76	1.78	46.02
rwelpc15	100.00	73.25	1.30
shall	99.81	99.84	99.94
beerpc	27.19	13.44	5.68

## Conclusions

This paper tests the computational stability of Donohue and Levitt's (2001) abortion-crime regressions. We use their original model specification and estimation methodology, new quality data on state-level crime correlates in the United States provided by Moody and Marvell (2010), and an algorithm for generating perturbed data sets proposed by Vinod (2009). Our results confirm the conclusions in previous studies that Donohue and Levitt's (2001) approach does not provide computationally stable regression coefficients, and therefore, their estimates of the abortion-crime relationship are unreliable.

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